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Calibration processes for photon-photon colliders

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Processes with creation of a pair charged particles with emission of hard photon and two pairs of charged particles are considered for colliding partially polarized photon photon beams. The effects of circular and linear polarization of the initial photons are discussed in more details.

The planned $\gamma\gamma$ colliding beams based on laser backward Compton scattering lepton high energy colliders [1] will provide a new laboratory for investigation of hadron properties. In [2] the general theory of polarization phenomena in colliding photon beams was developed. So a lot of attention was paid to the details of conversion of laser photons in the process of backward Compton scattering and to the effects of density distribution in the photon beams.

However, for the purposes of calibration and a measurement of the degree of polarization of photon beams the following QED processes with creation of one and two different pairs of leptons

$$\gamma(k_1) + \gamma(k_2) \to \mu^+(q_+) + \mu^-(q_-), \tag{1a}$$

$$\gamma(k_1) + \gamma(k_2) \to \mu^+(q_+) + \mu^-(q_-) + \gamma(k), \tag{1b}$$

$$\gamma(k_1) + \gamma(k_2) \to \bar{a}(p_+) + a(p_-) + \bar{b}(q_+) + b(q_-) \tag{1c}$$

can be utilized. Besides the latter, these processes provide an essential background for study of the hadron creation processes as well as ones with heavy vector bosons. It has to be stressed, that the polarization phenomena turns out to be essential in this analysis.

A lot of attention [1] was paid to the processes (1a, 1b) as well as to the process (1c) [3, 4]. In the one last especially the kinematics of the main contribution to the total cross section was discussed more carefully, namely when the final particles move in the narrow

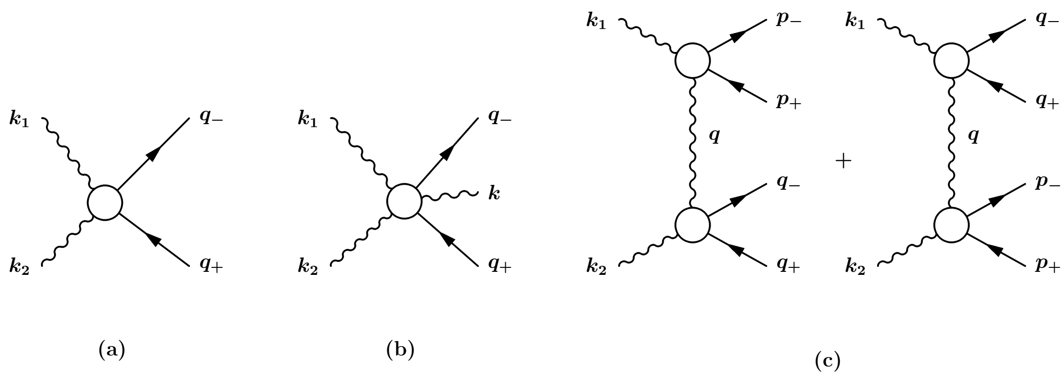

Fig. 1: Feynman diagram for the process $\gamma(k_1) + \gamma(k_2) \to \mu^+(q_+) + \mu^-(q_-)$ (a), the process $\gamma(k_1) + \gamma(k_2) \to \mu^+(q_+) + \mu^-(q_-) + \gamma(k)$ (b) and the process $\gamma(k_1) + \gamma(k_2) \to e^+(p_+) + e^-(p_-) + \mu^+(q_+) + \mu^-(q_-)$ (c).

cones along the direction of the photon colliding beams $\theta_i \sim (M_i/\sqrt{s})$, $s = 2k_1k_2$. Its total cross section do not decrease as a function of the total cms energy $\sqrt{s}$.

The single pair creation processes (1a, 1b) are considered in the kinematical region, when the hard final particles move in the large angles in cms ($\frac{m}{\sqrt{s}} \ll \theta_i \ll 1$), so the scalar products of all different 4-vectors are large compared with the square masses of particles. The process (1c) is investigated in the quasi peripherical kinematical region, where the invariant masses of produced pairs are much smaller than $\sqrt{s}$ and large compared with masses of particles. In our opinion this kinematics is useful in the experiments because of absence of phone processes. The rough estimation of differential cross sections is

$$\frac{\alpha^2}{s}, \ \frac{\alpha^3}{s}, \ \frac{\alpha^4}{s_{max}},$$

with $s_{max} = \max\{s_1, s_2\}$ and $s_{1,2}$ are the invariant masses squared of electron and muon pairs respectively. So in a kinematical situation, when $s_{max} \ll s$, the process (1c) is dominant.

The analysis of polarization phenomena in these all kinematical regions (which is absent to our knowledge up to now) is the motivation of our paper. Our paper is organized as follows: in part 1 we consider the processes (1a, 1b), in part 2 the process (1c). The polarization effects are studied by using definite expressions for the chiral amplitudes. We consider only the kinematics (mentioned above) the invariant mass of each pair is larger than masses of particles and smaller than cms total energy, moving preferably along the one

of initial photons.

Process $\gamma\gamma \to \mu^+\mu^-$ and $\gamma\gamma \to \mu^+\mu^-\gamma$.

The matrix elements of processes (1a, 1b) are (we put the masses of leptons to be equal to zero)

$$M^{\delta}_{\lambda_1\lambda_2} = -i4\pi\alpha\bar{u}_\delta(q_-)\left[\hat{\varepsilon}_{\lambda_1}\frac{\hat{q}_- - \hat{k}_1}{-\chi_{1-}}\hat{\varepsilon}_{\lambda_2} + \hat{\varepsilon}_{\lambda_2}\frac{-\hat{q}_+ + \hat{k}_1}{-\chi_{1+}}\hat{\varepsilon}_{\lambda_1}\right]v_\delta(q_+) \tag{2}$$

and

$$\begin{aligned} M^{\lambda\delta}_{\lambda_1\lambda_2} =& i(4\pi\alpha)^{3/2}\bar{u}_\delta(q_-) \\ &\times\left[\hat{\varepsilon}^*_\lambda\frac{\hat{q}_- + \hat{k}}{\chi_-}\hat{\varepsilon}_{\lambda_1}\frac{-\hat{q}_+ + \hat{k}_2}{-\chi_{2+}}\hat{\varepsilon}_{\lambda_2} + \hat{\varepsilon}_{\lambda_1}\frac{\hat{q}_- - \hat{k}_1}{-\chi_{1-}}\hat{\varepsilon}^*_\lambda\frac{-\hat{q}_+ + \hat{k}_2}{-\chi_{2+}}\hat{\varepsilon}_{\lambda_2}\right. \\ &+ \hat{\varepsilon}_{\lambda_1}\frac{\hat{q}_- - \hat{k}_1}{-\chi_{1-}}\hat{\varepsilon}_{\lambda_2}\frac{-\hat{q}_+ - \hat{k}}{\chi_+}\hat{\varepsilon}^*_\lambda + \hat{\varepsilon}^*_\lambda\frac{\hat{q}_- + \hat{k}}{\chi_-}\hat{\varepsilon}_{\lambda_2}\frac{-\hat{q}_+ + \hat{k}_1}{-\chi_{1+}}\hat{\varepsilon}_{\lambda_1} \\ &\left.+ \hat{\varepsilon}_{\lambda_2}\frac{\hat{q}_- - \hat{k}_2}{-\chi_{2-}}\hat{\varepsilon}^*_\lambda\frac{-\hat{q}_+ + \hat{k}_1}{-\chi_{1+}}\hat{\varepsilon}_{\lambda_1} + \hat{\varepsilon}_{\lambda_2}\frac{\hat{q}_- - \hat{k}_2}{-\chi_{2-}}\hat{\varepsilon}_{\lambda_1}\frac{-\hat{q}_+ - \hat{k}}{\chi_+}\hat{\varepsilon}^*_\lambda\right]v_\delta(q_+), \end{aligned} \tag{3}$$

with

$$\chi_{1\pm} = 2k_1q_\pm\,, \quad \chi_{2\pm} = 2k_2q_\pm\,, \quad \chi_\pm = 2kq_\pm \quad \lambda, \lambda_1, \lambda_2, \delta = \pm 1, \tag{4}$$

and $\hat{\varepsilon}_\lambda$, $\bar{u}_\delta$, v_δ describe the definite chiral states of photons

$$\begin{gathered} \hat{\varepsilon}_\lambda(k) = N[\hat{q}_-\hat{q}_+\hat{k}\omega_{-\lambda} - \hat{k}\hat{q}_-\hat{q}_+\omega_\lambda]\,, \quad N = [s_1\chi_-\chi_+/2]^{-1/2}\,, \\ \hat{\varepsilon}_{\lambda_1}(k_1) = N_1[\hat{q}_-\hat{q}_+\hat{k}_1\omega_{-\lambda_1} - \hat{k}_1\hat{q}_-\hat{q}_+\omega_{\lambda_1}]\,, \quad N_1 = [s_1\chi_{1-}\chi_{1+}/2]^{-1/2}\,, \\ \hat{\varepsilon}_{\lambda_2}(k_2) = N_2[\hat{q}_-\hat{q}_+\hat{k}_2\omega_{-\lambda_2} - \hat{k}_2\hat{q}_-\hat{q}_+\omega_{\lambda_2}]\,,\ N_2 = [s_1\chi_{2-}\chi_{2+}/2]^{-1/2}\,, \\ s_1 = 2q_-q_+\,, \quad \omega_{\pm\lambda} = (1 \pm \lambda\gamma_5)/2 \end{gathered} \tag{5}$$

and fermions [6, 8]

$$u_\delta = \omega_\delta u, \quad v_\delta = \omega_{-\delta}v, \quad \bar{u}_\delta = \bar{u}\omega_{-\delta}, \quad \bar{v}_\delta = \bar{v}\omega_\delta\,. \tag{6}$$

For the process (1a) the chiral amplitudes $M^{\delta}_{\lambda_1\lambda_2}$ are (we omit $i(\sqrt{4\pi\alpha})^n$ factor)

$$\begin{aligned} M^{\pm}_{+-} &= \mp N_1N_2s\chi_{2\mp}\bar{u}(q_-)\hat{k}_1\omega_\mp v(q_+), \\ M^{\pm}_{-+} &= \pm N_1N_2s\chi_{2\pm}\bar{u}(q_-)\hat{k}_1\omega_\mp v(q_+)\,, \end{aligned} \tag{7}$$

chiral amplitudes with equal chiralities of photons are equal zero.

For a calculating of the cross sections for processes (1a-1c) in the case of partially polarized photon beams with momentum $k_i\,(i=1,2)$ we will use polarization matrices of density $\rho_i = \rho_i(k_i)$ in the helically representation determined by Stokes parameters $\vec{\xi}^{(i)}$ in the following way [9]

$$\rho_i = \rho_i(k_i) \;=\; \frac{1}{2}\begin{pmatrix} 1+\xi_2^{(i)} & i\,\xi_1^{(i)} - \xi_3^{(i)} \\ -i\,\xi_1^{(i)} - \xi_3^{(i)} & 1-\xi_2^{(i)} \end{pmatrix}, \;\mathrm{Tr}\,(\rho_i) = 1. \tag{8}$$

Let us introduce 2×2 matrix building from the amplitudes (7)

$$\mathcal{M}_1^\delta = \begin{pmatrix} M_{++}^\delta & M_{+-}^\delta \\ M_{-+}^\delta & M_{--}^\delta \end{pmatrix}. \tag{9}$$

Than the probability of the process (1a) will be reduce to calculation of trace from the product of the next matrices [2]

$$|M_{\lambda_1\lambda_2}^\delta|^2 \to \mathrm{Tr}\,(\rho_1^T\,\mathcal{M}_1^\delta\,\rho_2\,\mathcal{M}_1^{\delta\dagger})\,, \tag{10}$$

where ρ_1^T is the matrix transposed to ρ_1 and $\mathcal{M}_1^\dagger$ is hermitian conjugated matrix to $\mathcal{M}_1$.

The cross section in general case has a form

$$\frac{\mathrm{d}\sigma_{\vec{\xi}_1\vec{\xi}_2\delta}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\alpha^2}{4s}\Big\{(1-\xi_2^{(1)}\xi_2^{(2)})R_+ -\, 2\,(\xi_1^{(1)}\xi_1^{(2)} + \xi_3^{(1)}\xi_3^{(2)}) + \delta(\xi_2^{(2)} - \xi_2^{(1)})R_-\Big\}, \tag{11}$$

where

$$R_\pm = \frac{\chi_{1+}^2 \pm \chi_{1-}^2}{\chi_{1-}\chi_{1+}}\;,\;\chi_{1\pm} = \chi_{2\mp}\,. \tag{12}$$

For the case of completely circularly polarized photons (right (R): $\xi_2^{(1,2)} = +1$, left (L): $\xi_2^{(1,2)} = -1$) we will have

$$\frac{\mathrm{d}\sigma_{LL}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\mathrm{d}\sigma_{RR}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = 0\,,\quad \frac{\mathrm{d}\sigma_{LR}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\mathrm{d}\sigma_{RL}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\alpha^2}{s}R_+\,.$$

Further when the colliding beams have equal or reverse complete linear polarization $\xi_1^{(1)} = \pm\xi_1^{(2)} = \pm 1$ ($\xi_3^{(1)} = \pm\xi_3^{(2)} = \pm 1$) from (11) we have

$$\frac{\mathrm{d}\sigma_{\pm\pm}^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\alpha^2}{2s}\,(R_+ \mp 2)\;.$$

Finally in the case of unpolarized particles we have the result which coincides with [9] in massless limit

$$\frac{\mathrm{d}\sigma^{\gamma\gamma\to\mu\bar{\mu}}}{\mathrm{d}\Omega_{\mu_-}} = \frac{\alpha^2}{2s}R_+\,.$$

For the process (1b) chiral amplitudes (3) could be written in the form

$$
\begin{aligned}
M_{++}^{-+} &= NN_1N_2s_1^2\chi_+\bar{u}\hat{k}\omega_+v, & M_{--}^{+-} &= NN_1N_2s_1^2\chi_+\bar{u}\hat{k}\omega_-v,\\
M_{+-}^{++} &= NN_1N_2s_1^2\chi_{2+}\bar{u}\hat{k}_2\omega_+v, & M_{-+}^{--} &= NN_1N_2s_1^2\chi_{2+}\bar{u}\hat{k}_2\omega_-v,\\
M_{-+}^{++} &= NN_1N_2s_1^2\chi_{1+}\bar{u}\hat{k}_1\omega_+v, & M_{+-}^{--} &= NN_1N_2s_1^2\chi_{1+}\bar{u}\hat{k}_1\omega_-v,\\
M_{-+}^{-+} &= -NN_1N_2s_1^2\chi_{2-}\bar{u}\hat{k}_2\omega_+v, & M_{+-}^{+-} &= -NN_1N_2s_1^2\chi_{2-}\bar{u}\hat{k}_2\omega_-v,\\
M_{+-}^{-+} &= -NN_1N_2s_1^2\chi_{1-}\bar{u}\hat{k}_1\omega_+v, & M_{-+}^{+-} &= -NN_1N_2s_1^2\chi_{1-}\bar{u}\hat{k}_1\omega_-v,\\
M_{--}^{++} &= -NN_1N_2s_1^2\chi_{-}\bar{u}\hat{k}\omega_+v, & M_{++}^{--} &= -NN_1N_2s_1^2\chi_{-}\bar{u}\hat{k}\omega_-v.
\end{aligned}
\tag{13}
$$

The matrix element squared for the process (1b) in the case of partially polarized initial beams and summed over the polarizations of final particles is calculated analogously to the process (1a) (see (10)). As a result for differential cross section with taken into account only polarization of photon beams we have

$$
\begin{aligned}
\mathrm{d}\sigma_{\vec{\xi}_1\vec{\xi}_2}^{\gamma\gamma\to\mu\bar{\mu}\gamma} &= \frac{\alpha^3 s_1}{2\pi^2 s}\,\frac{T_{in}}{D}\,\mathrm{d}\Gamma\,, \quad D = \chi_-\chi_+\chi_{1-}\chi_{1+}\chi_{2-}\chi_{2+}\,,\\
\mathrm{d}\Gamma &= \frac{\mathrm{d}^3 q_+}{\epsilon_+}\frac{\mathrm{d}^3 q_-}{\epsilon_-}\frac{\mathrm{d}^3 k}{\omega}\delta^4(k_1+k_2-q_+-q_--k)\,,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
T_{in} = {} & \big(\xi_1^{(1)}\xi_1^{(2)}+\xi_3^{(1)}\xi_3^{(2)}\big)(\chi_{1+}\chi_{2+}+\chi_{1-}\chi_{2-})(\chi_+\chi_- - \chi_{1+}\chi_{1-} - \chi_{2+}\chi_{2-})\\
& + 4\big(\xi_1^{(1)}\xi_3^{(2)}-\xi_3^{(1)}\xi_1^{(2)}\big)(\chi_{1+}\chi_{2+}-\chi_{1-}\chi_{2-})\,E_q\\
& - 4\xi_1^{(1)}(\chi_+\chi_{2-}-\chi_-\chi_{2+})\,E_q + 4\xi_1^{(2)}(\chi_+\chi_{1-}-\chi_-\chi_{1+})\,E_q\\
& + \xi_3^{(1)}(\chi_+\chi_{2-}+\chi_-\chi_{2+})(\chi_+\chi_- - \chi_{1+}\chi_{1-} + \chi_{2+}\chi_{2-})\\
& + \xi_3^{(2)}(\chi_+\chi_{1-}+\chi_-\chi_{1+})(\chi_+\chi_- + \chi_{1+}\chi_{1-} - \chi_{2+}\chi_{2-})\\
& + \xi_2^{(1)}\xi_2^{(2)}\big[\chi_+\chi_-(\chi_+^2+\chi_-^2) - \chi_{1+}\chi_{1-}(\chi_{1+}^2+\chi_{1-}^2) - \chi_{2+}\chi_{2-}(\chi_{2+}^2+\chi_{2-}^2)\big]\\
& + \chi_+\chi_-(\chi_+^2+\chi_-^2) + \chi_{1+}\chi_{1-}(\chi_{1+}^2+\chi_{1+}^2) + \chi_{2+}\chi_{2-}(\chi_{2+}^2+\chi_{2-}^2)\,,
\end{aligned}
$$

where $E_q = \epsilon_{\mu\nu\rho\sigma}k_1^\mu k_2^\nu q_+^\rho q_-^\sigma. = \frac{s}{2}[\boldsymbol{q}_+\boldsymbol{q}_-]_z$.

The corresponding cross sections for definite chiral states of initial photons are

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{RR(LL)}^{\gamma\gamma\to\mu^-\mu^+\gamma}}{\mathrm{d}\Gamma} &= \frac{\alpha^3 s_1}{\pi^2 s}\;\frac{\chi_-\chi_+(\chi_-^2+\chi_+^2)}{D}\,,\\
\frac{\mathrm{d}\sigma_{RL(LR)}^{\gamma\gamma\to\mu^-\mu^+\gamma}}{\mathrm{d}\Gamma} &= \frac{\alpha^3 s_1}{\pi^2 s}\;\frac{\chi_{1-}\chi_{1+}(\chi_{1-}^2+\chi_{1+}^2)+\chi_{2-}\chi_{2+}(\chi_{2-}^2+\chi_{2+}^2)}{D}\,.
\end{aligned}
\tag{15}
$$

For the case when only one photon beam polarized we gained the cross section similar to (14) with the replacement $T_{in} \to T_{in}^{(1)}$, where

$$
\begin{aligned}
T_{in}^{(1)} =& \chi_+\chi_-(\chi_+^2+\chi_-^2)+\chi_{1+}\chi_{1-}(\chi_{1+}^2+\chi_{1+}^2) \\
&+\chi_{2+}\chi_{2-}(\chi_{2+}^2+\chi_{2-}^2)-4\xi_1^{(1)}(\chi_+\chi_{2-}-\chi_-\chi_{2+})\,E_q \\
&+\xi_3^{(1)}(\chi_+\chi_{2-}+\chi_-\chi_{2+})(\chi_+\chi_--\chi_{1+}\chi_{1-}+\chi_{2+}\chi_{2-})\;.
\end{aligned}
\tag{16}
$$

In the case when all particles are unpolarized the differential cross section is

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_0^{\gamma\gamma\to\mu^-\mu^+\gamma}}{\mathrm{d}\Gamma} =& \frac{\alpha^3 s_1}{2\pi^2 s}\Big[\chi_-\chi_+(\chi_-^2+\chi_+^2) \\
&+\chi_{1-}\chi_{1+}(\chi_{1-}^2+\chi_{1+}^2)+\chi_{2-}\chi_{2+}(\chi_{2-}^2+\chi_{2+}^2)\Big]/D\;.
\end{aligned}
\tag{17}
$$

The probability of the process (1b) in the case when initial and emitted photons with momenta k_1 and k are partially polarized with Stokes parameters $\vec{\xi}^{(1)}$ and $\vec{\xi}$ is calculated analogously to (10).

Summing over the polarizations of final μ^+ and μ^- particles one can get the differential cross section which takes into account polarization of photons with momentum k_1 and k

$$
\mathrm{d}\sigma_{\vec{\xi}_1\vec{\xi}}^{\gamma\gamma\to\mu\bar{\mu}\gamma} = \frac{\alpha^3 s_1}{4\pi^2 s}\,\frac{T_{fin}}{D}\,\mathrm{d}\Gamma\;,
\tag{18}
$$

$$
\begin{aligned}
T_{fin} =& \big(\xi_1^{(1)}\xi_1+\xi_3^{(1)}\xi_3\big)(\chi_+\chi_{1+}+\chi_-\chi_{1-})(\chi_+\chi_-+\chi_{1+}\chi_{1-}-\chi_{2+}\chi_{2-}) \\
&+4(\xi_1^{(1)}\xi_3-\xi_3^{(1)}\xi_1)(\chi_+\chi_{1+}-\chi_-\chi_{1-})\,E_q \\
&-4\xi_1^{(1)}(\chi_+\chi_{2-}-\chi_-\chi_{2+})\,E_q-4\xi_1(\chi_{1+}\chi_{2-}-\chi_{1-}\chi_{2+})\,E_q \\
&+\xi_3^{(1)}(\chi_+\chi_{2-}+\chi_-\chi_{2+})(\chi_+\chi_--\chi_{1+}\chi_{1-}+\chi_{2+}\chi_{2-}) \\
&+\xi_3(\chi_{1+}\chi_{2-}+\chi_{1-}\chi_{2+})(\chi_+\chi_--\chi_{1+}\chi_{1-}-\chi_{2+}\chi_{2-}) \\
&+\xi_2^{(1)}\xi_2\big[\chi_+\chi_-(\chi_+^2+\chi_-^2)+\chi_{1+}\chi_{1-}(\chi_{1+}^2+\chi_{1-}^2)-\chi_{2+}\chi_{2-}(\chi_{2+}^2+\chi_{2-}^2)\big] \\
&+\chi_+\chi_-(\chi_+^2+\chi_-^2)+\chi_{1+}\chi_{1-}(\chi_{1+}^2+\chi_{1-}^2)+\chi_{2+}\chi_{2-}(\chi_{2+}^2+\chi_{2-}^2)\;.
\end{aligned}
\tag{19}
$$

Obtained expressions (18, 19) allow us to determine Stokes parameters of emitted photon versus polarization of initial photon $\vec{\xi}^{(1)}$ with momentum k_1

$$\xi_1^f = \frac{f_{10} + \xi_1^{(1)} f_{11} + \xi_3^{(1)} f_{13}}{f_{00} + \xi_1^{(1)} f_{01} + \xi_3^{(1)} f_{03}} \,, \quad \xi_3^f = \frac{f_{30} + \xi_1^{(1)} f_{31} + \xi_3^{(1)} f_{33}}{f_{00} + \xi_1^{(1)} f_{01} + \xi_3^{(1)} f_{03}} \,, \tag{20}$$
$$\xi_2^f = \frac{\xi_2^{(1)} f_{22}}{f_{00} + \xi_1^{(1)} f_{01} + \xi_3^{(1)} f_{03}} \,,$$

where

$$\begin{aligned}
f_{00} &= \chi_+\chi_-(\chi_+^2 + \chi_-^2) + \chi_{1+}\chi_{1-}(\chi_{1+}^2 + \chi_{1-}^2) + \chi_{2+}\chi_{2-}(\chi_{2+}^2 + \chi_{2-}^2), \\
f_{01} &= -4(\chi_+\chi_{2-} - \chi_-\chi_{2+})\, E_q, \\
f_{03} &= (\chi_+\chi_{2-} + \chi_-\chi_{2+})(\chi_+\chi_- - \chi_{1+}\chi_{1-} + \chi_{2+}\chi_{2-}), \\
f_{10} &= -4(\chi_{1+}\chi_{2-} - \chi_{1-}\chi_{2+})\, E_q, \\
f_{11} &= (\chi_+\chi_{1+} + \chi_-\chi_{1-})(\chi_+\chi_- + \chi_{1+}\chi_{1-} - \chi_{2+}\chi_{2-}), \\
f_{13} &= -4(\chi_+\chi_{1+} - \chi_-\chi_{1-})\, E_q, \\
f_{22} &= \chi_+\chi_-(\chi_+^2 + \chi_-^2) + \chi_{1+}\chi_{1-}(\chi_{1+}^2 + \chi_{1-}^2) - \chi_{2+}\chi_{2-}(\chi_{2+}^2 + \chi_{2-}^2), \\
f_{30} &= (\chi_{1+}\chi_{2-} + \chi_{1-}\chi_{2+})(\chi_+\chi_- - \chi_{1+}\chi_{1-} - \chi_{2+}\chi_{2-}), \\
f_{31} &= -f_{13} = 4(\chi_+\chi_{1+} - \chi_-\chi_{1-})\, E_q, \\
f_{33} &= f_{11} = (\chi_+\chi_{1+} + \chi_-\chi_{1-})(\chi_+\chi_- + \chi_{1+}\chi_{1-} - \chi_{2+}\chi_{2-})\,.
\end{aligned} \tag{21}$$

Process $\gamma\gamma \to a(p_-)\bar{a}(p_+)b(q_-)\bar{b}(q_+)$**.**

The kinematical variables of two pairs production process in quasi peripherical photon collisions are defined as (see (1c))

$$2k_1k_2 = s, \quad (p_+ + p_-)^2 = s_1\,, \quad (q_+ + q_-)^2 = s_2\,, \tag{22}$$
$$q = k_1 - p_+ - p_- = q_+ + q_- - k_2\,, \quad \chi_\pm = 2k_1p_\pm\,, \quad \chi'_\pm = 2k_2q_\pm\,.$$

Further we will consider the case when the e^+e^- pair moves in the same hemisphere with the photon of the momentum k_1 and the muon pair moves with another photon in an opposite hemisphere. Besides the latter we will restrict ourselves by the case when the invariant masses e^+e^- and $\mu^+\mu^-$ are large in comparison with the mass of muon and much less than the center of mass total energy $\sqrt{s}$

$$m_\mu^2 \ll s_1, \quad s_2 \ll s, \quad \chi_\pm \sim s_1, \quad \chi'_\pm \sim s_2. \tag{23}$$

A contribution of this region will not depend on s at large s, and will be dominant. For this kinematics the components of the created pairs move inside the cones with polar angle of order $\theta_{1,2} \sim \sqrt{s_{1,2}/s}$ along the beam axes. A few Feynman amplitudes are relevant in this kinematics, which can be drawn in the form of two block (see Fig. 1). The corresponding matrix element has a factorized form

$$M(\gamma\gamma \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^-) = is\frac{2(4\pi\alpha)^2}{q^2}\left[m_1^{\lambda_1\lambda_e}m_2^{\lambda_2\lambda_\mu} + m_1^{\lambda_2\lambda_e}m_2^{\lambda_1\lambda_\mu}\right], \tag{24}$$
$$m_1^{\lambda_1\lambda_e} = \frac{1}{s}k_2^\mu\varepsilon_1^\nu(k_1)M_{1\mu\nu}, \quad m_2^{\lambda_2\lambda_\mu} = \frac{1}{s}k_1^\sigma\varepsilon_2^\rho(k_2)M_{2\sigma\rho}.$$

In this two back-to-back kinematical regions the Sudakov parametrization for 4-momenta is convenient [5]. Assuming that the pair $a(p_-)\bar{a}(p_+)$ belongs to the jet moving along direction of photon k_1 we have

$$p_\pm = \alpha_\pm k_2 + x_\pm k_1 + p_\pm^\perp, \quad (p_\pm^\perp)^2 = -\boldsymbol{p}_\pm^2, \quad p_\pm^\perp.k_1 = p_\pm^\perp.k_2 = 0, \tag{25}$$
$$\alpha_\pm \approx \frac{\chi_{1\pm}}{s} = \frac{\boldsymbol{p}_\pm^2}{sx_\pm}, \quad s_1 = (p_+ + p_-)^2 = \frac{\boldsymbol{Q_a}^2}{x_+x_-}, \quad \boldsymbol{Q_a} = x_+\boldsymbol{p}_- - x_-\boldsymbol{p}_+,$$

where $\boldsymbol{p}_\pm$ are 2-dimensional euclidean vectors ($\boldsymbol{p}_- + \boldsymbol{p}_+ + \boldsymbol{q} = 0$), $x_\pm = 2k_2p_\pm/s$ are energy fractions of pair components ($x_+ + x_- = 1$) and s_1 is squared invariant mass of the pair.

The similar relations are valid for the pair $b(q_-)\,\bar{b}(q_+)$ from the opposite jet moving in the direction of the photon k_2

$$q_\pm = y_\pm k_2 + \beta_\pm k_1 + q_\pm^\perp\,, \quad q_\pm^\perp = -\boldsymbol{q}_\pm^2\,, \quad q_\pm^\perp.k_1 = q_\pm^\perp.k_2 = 0\,, \tag{26}$$
$$\beta_\pm \approx \frac{\chi'_{2\pm}}{s} = \frac{\boldsymbol{q}_\pm^2}{sy_\pm}\,, \quad s_2 = (q_+ + q_-)^2 = \frac{\boldsymbol{Q_b}^2}{y_+y_-}\,, \quad \boldsymbol{Q_b} = y_+\boldsymbol{q}_- - y_-\boldsymbol{q}_+\,,$$

where energy fractions $y_\pm = 2k_1q_\pm/s$, ($y_+ + y_- = 1$) and $\boldsymbol{q}_- + \boldsymbol{q}_+ - \boldsymbol{q} = 0$.

We define the chiral amplitudes as a matrix elements calculated with definite chiral states of fermions and photons. We choice the polarization vectors of photons in the form [6]

$$\varepsilon_\mu^{\lambda_j}(k) = \varepsilon_\mu^{||} + i\lambda_j\varepsilon_\mu^\perp \tag{27}$$

$$\varepsilon_\mu^{||} = N_2[(q_-k)q_{+\mu} - (q_+k)q_{-\mu}]\,, \quad \varepsilon_\mu^\perp = N_2\epsilon_{\mu\alpha\beta\gamma}q_-^\alpha q_+^\beta k^\gamma\,,$$
$$\hat{\varepsilon}^{\lambda_1} = N_1[\hat{p}_-\hat{p}_+\hat{k}_1\omega_{-\lambda_1} - \hat{k}_1\hat{p}_-\hat{p}_+\omega_{\lambda_1}]\,, \quad N_1 = [s_1\chi_+\chi_-/2]^{-1/2}\,,$$
$$\hat{\varepsilon}^{\lambda_2} = N_2[\hat{q}_-\hat{q}_+\hat{k}_2\omega_{-\lambda_2} - \hat{k}_2\hat{q}_-\hat{q}_+\omega_{\lambda_2}]\,, \quad N_2 = [s_2\chi'_+\chi'_-/2]^{-1/2}\,.$$

Chiral states of fermions were defined above.

In derivation of cross section we consider only first term in (24), the second one can be obtained by correspondent replacement. Their interference term vanish in limit $s \to \infty$ and is desregarded.

$$M^{\lambda_1\lambda_e\lambda_2\lambda_\mu} = is\frac{2(4\pi\alpha)^2}{\boldsymbol{q}^2} m_1^{\lambda_1\lambda_e} m_2^{\lambda_2\lambda_\mu},\ \lambda_j = \pm 1,\ j = 1, 2, e, \mu. \tag{28}$$

The quantities $m_1^{\lambda_1\lambda_e}$ for lepton pair have a form

$$m_1^{\lambda_1\lambda_e} = \frac{N_1}{s}\bar{u}(p_-)\left[\delta_{\lambda_e\lambda_1}\hat{p}_+\hat{q}\hat{k}_2 + \delta_{\lambda_e(-\lambda_1)}\hat{k}_2\hat{q}\hat{p}_-\right]\omega_{\lambda_1}v(p_+), \tag{29}$$

For the case of creation of charged pion pair $\pi^+\pi^-$ one gets

$$m_1^\lambda = -N_2\big\{\boldsymbol{Q_a}\cdot\boldsymbol{q} - i\lambda[\boldsymbol{Q_a}, \boldsymbol{q}]_z\big\}. \tag{30}$$

Quantities $m_2^{\lambda_2\lambda_\mu}$ are constructed analogically.

Let us introduce 2×2 matrices building from the amplitudes (29)

$$\mathcal{M}_1^{\lambda_e} = \begin{pmatrix} m_+m_+^* & m_+m_-^* \\ m_-m_+^* & m_-m_-^* \end{pmatrix}, \quad m_\pm = m_1^{\lambda_1\lambda_e} \ \ (\lambda_1 = \pm 1), \tag{31a}$$

$$\mathcal{M}_2^{\lambda_\mu} = \begin{pmatrix} m'_+{m'_+}^* & m'_+{m'_-}^* \\ m'_-{m'_+}^* & m'_-{m'_-}^* \end{pmatrix}, \quad m'_\pm = m_2^{\lambda_2\lambda_\mu} \ \ (\lambda_2 = \pm 1), \tag{31b}$$

then the absolute values of matrix elements squared of the process (1c) will be reduce to calculation of a trace from the product of the matrices as follows

$$\left|m_1^{\lambda_1\lambda_e}\right|^2 = \mathrm{Tr}\,(\rho_1\,\mathcal{M}_1^{\lambda_e}), \quad \left|m_2^{\lambda_2\lambda_\mu}\right|^2 = \mathrm{Tr}\,(\rho_2\,\mathcal{M}_2^{\lambda_\mu})\,. \tag{32}$$

A simple calculation leads to the result

$$\mathcal{M}_{1(e)}^{\pm} = N_1^2\boldsymbol{Q}_{\boldsymbol{a}}^2\boldsymbol{q}^2 \begin{pmatrix} x_\pm/x_\mp & \exp\{\pm 2i\varphi_a\} \\ \exp\{\mp 2i\varphi_a\} & x_\mp/x_\pm \end{pmatrix}, \tag{33a}$$

$$\mathcal{M}_{1(\pi)} = N_1^2\boldsymbol{Q}_{\boldsymbol{a}}^2\boldsymbol{q}^2 \begin{pmatrix} 1 & \exp\{2i\varphi_a\} \\ \exp\{-2i\varphi_a\} & 1 \end{pmatrix}, \quad \varphi_a = \widehat{\boldsymbol{Q_a}\boldsymbol{q}}. \tag{33b}$$

Let us now transform the final state phase space volume

$$\begin{aligned} \mathrm{d}\Phi_4 &= \frac{\mathrm{d}^3p_+}{2p_{+0}}\frac{\mathrm{d}^3p_-}{2p_{-0}}\frac{\mathrm{d}^3q_+}{2q_{+0}}\frac{\mathrm{d}^3q_-}{2q_{-0}}\,\delta^4(k_1 + k_2 - q_+ - q_- - p_+ - p_-) \\ &= \mathrm{d}^4q\,\mathrm{d}^4p_+\,\mathrm{d}^4p_-\,\mathrm{d}^4q_+\,\mathrm{d}^4q_-\,\delta^4(k_1 + q - p_+ - p_-)\delta^4(k_2 - q - q_+ - q_-) \\ &\quad\times\delta(p_+^2)\,\delta(p_-)\,\delta(q_+^2)\,\delta(q_-^2). \end{aligned} \tag{34}$$

Applying Sudakov representation ($\mathrm{d}^4q = \frac{s}{2}\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}^2\boldsymbol{q}$) one can rewrite (34) into the following form

$$\mathrm{d}\Phi_4 = \frac{\mathrm{d}x_-\,\mathrm{d}y_-\,\mathrm{d}^2q_+\,\mathrm{d}^2q_-\,\mathrm{d}^2q}{8s\,x_+\,x_-\,y_+\,y_-}. \tag{35}$$

For the cross section of two different pairs production (i.e., $a \neq b$) one obtains

$$\begin{aligned}\mathrm{d}\sigma(\gamma\gamma \to a\bar{a}b\bar{b}) =& \frac{\alpha^4}{\pi}x_+x_-y_+y_-\frac{\mathrm{d}x_-\mathrm{d}y_-\mathrm{d}^2q_-\mathrm{d}^2p_-\mathrm{d}^2q}{\boldsymbol{p}_-^2\boldsymbol{q}_-^2} \\ &\times\left[\frac{S_{1(a)}S_{2(b)}}{(\boldsymbol{q}+\boldsymbol{q}_-)^2(\boldsymbol{q}-\boldsymbol{p}_-)^2}+\frac{S_{1(b)}S_{2(a)}}{(\boldsymbol{q}-\boldsymbol{q}_-)^2(\boldsymbol{q}+\boldsymbol{p}_-)^2}\right],\end{aligned} \tag{36}$$

and for the case $a = b$

$$\begin{aligned}\mathrm{d}\sigma(\gamma\gamma \to a\bar{a}a\bar{a}) =& \frac{\alpha^4}{\pi}x_+x_-y_+y_-\frac{\mathrm{d}x_-\mathrm{d}y_-\mathrm{d}^2q_-\mathrm{d}^2p_-\mathrm{d}^2q}{\boldsymbol{p}_-^2\boldsymbol{q}_-^2} \\ &\times\frac{S_{1(a)}S_{2(a)}}{(\boldsymbol{q}+\boldsymbol{q}_-)^2(\boldsymbol{q}-\boldsymbol{p}_-)^2}\end{aligned} \tag{37}$$

where

$$S_{1(a)} = \frac{\mathrm{Tr}\,(\rho_1\,\mathcal{M}_{1(a)})}{\boldsymbol{q}^2\boldsymbol{Q}^2N_1^2}\,, \quad S_{2(a)} = \frac{\mathrm{Tr}\,(\rho_2\,\mathcal{M}_{2(a)})}{\boldsymbol{q}^2\boldsymbol{Q}^2N_2^2}\,. \tag{38}$$

Explicit form of (38) is

$$\begin{aligned}S_{1(\pi)} &= 1-\xi_3^{(1)}\cos 2\varphi_a+\xi_1^{(1)}\sin 2\varphi_a\,,\\ S_{2(\pi)} &= 1-\xi_3^{(2)}\cos 2\varphi_b-\xi_1^{(2)}\sin 2\varphi_b\,,\\ S_{1(\mu)} &= \frac{x_-^2+x_+^2}{2x_+x_-}-\lambda_\mu\xi_2^{(1)}\frac{x_--x_+}{2x_+x_-}-\xi_3^{(1)}\cos 2\varphi_a+\xi_1^{(1)}\sin 2\varphi_a\,,\\ S_{2(\mu)} &= \frac{y_-^2+y_+^2}{2y_+y_-}-\lambda_\mu\xi_2^{(2)}\frac{y_--y_+}{2y_+y_-}-\xi_3^{(2)}\cos 2\varphi_b+\xi_1^{(2)}\sin 2\varphi_b\,,\end{aligned} \tag{39}$$

with angles $\varphi_j = \widehat{\boldsymbol{Q_j q}}$.

In conclusion we note that the cross section of two pair production in kinematical region, when all hard particles move in large angles even in unpolarized case, has a very complicated form (see for example the paper [7], where it was obtained for a cross channel). The ratio of magnitudes of cross sections in peripherical kinematics to the last one is $\frac{s}{s_{max}} \gg 1$, which underline the importance of quasi peripherical kinematics, considered in this paper.

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